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G^2 Curve Design with Generalised Cornu Spiral

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ABSTRACT

This paper described the application of Generalised Cornu Spiral (GCS) in aesthetic design. The aim of using GCS in aesthetic design is because of it has the excellent curvature properties – the rational linear curvature profile. GCS is a transcendental function. Thus it is important to approximate the GCS by polynomial curve. The approximation is described in the recent work. This paper described the application of GCS in curve design and is illustrated with vase and glass designs.

Keywords: GCS, fair curve.

INTRODUCTION

Fair curves are always desirable in aesthetic design. Although the concept of fairness is rather fuzzy, most of the engineers agree that fairness of curve can be measured base on its curvature profile. Farin (1989) had discussed about the usage of curvature in measuring the fairness of curve. Burchard (1994) had discussed about the relation between aesthetic and monotonicity of curvature.

Spiral is the engineer's favorite curves not only because of its beautiful visual appearance but also because of its monotone curvature profile. Among all spirals, Cornu spiral is well known by its linear curvature profile. It is widely applied in engineering design. Application of Cornu spiral in previous literature included highway and rail way design, sketch-based road design, and consumer product designs (McCrae and Singh (2009), Muratov (2002), Walton and Meek (1998, 2007)).

GCS is a family of spirals which can be reduced to Cornu spiral, logarithmic spiral, circle and straight line. It has the rational linear curvature profile and thus it is suitable to apply in fair curve design. The objective of this paper is to describe the application of GCS in curve design for aesthetic product. The rest of the paper is organized as following. Section 2 introduce of Cornu spiral. Section 3 introduce of GCS. Section 4 discusses the application of GCS in vase and glass design. Section 5 illustrates the results of discussion.

CORNU SPIRAL

Cornu Spiral also referred as Euler spiral and clothoid was first investigated by Leonhard Euler in 17 century. A sample of Cornu spiral is show in figure 1. Cornu spiral is well known as the elegant curve with linear curvature profile. This advanced curvature property made Cornu spiral the perfect adequate transition curve for connecting the circles and straight lines in railway and highway design.

Referred to Meek and Walton(1992), the parametric equation of Cornu spiral is defined in term of Fresnel integral as

$$x(t) = a\sqrt{\pi}\, FresnelS\left[\frac{t}{\sqrt{\pi}}\right], \quad y(t) = a\sqrt{\pi}\, FresnelC\left[\frac{t}{\sqrt{\pi}}\right], \tag{1}$$

where $FresnelS[u] = \int_0^u \sin\left[\frac{\pi t^2}{2}\right]dt$, $FresnelS[u] = \int_0^u \cos\left[\frac{\pi t^2}{2}\right]dt$ and a is the scaling parameter. The curvature of Cornu spiral is defined as

$$k(t) = \frac{t}{a} \tag{2}$$

Curvature profile of Cornu spiral is showed in figure 2.

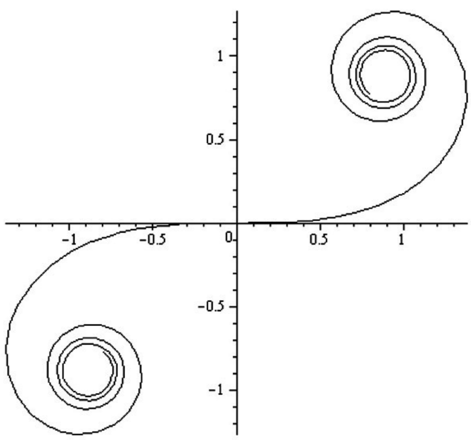


Figure 1. Cornu Spira

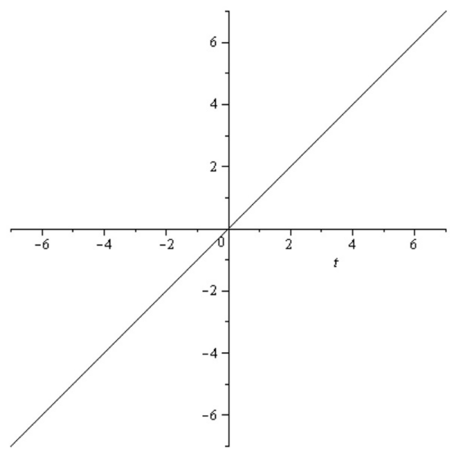


Figure 2. Curvature profile of Cornu spiral

GENERALISED CORNU SPIRAL (GCS)

GCS is a family of spirals introduced by Ali *et al*. (1999). The GCS has rational linear curvature profile. It can be reduced to Cornu Spiral, Logarithmic spiral, circle and straight line by controlling the free parameters in its curvature function. Another advantage of GCS is that its curvature can be adjusted by varying the shape factor in curvature function. The parametric equation GCS can be defined in term of arc length as

$$x(s) = x(0) + \int_0^s \cos\left\{\theta(0) + \int_0^t k(u)du\right\}dt, \; y(s) = y(0) + \int_0^s \sin\left\{\theta(0) + \int_0^t k(u)du\right\}dt \tag{3}$$

The curvature of GCS is a function of arc length defined as

$$k(s) = \frac{p + qs}{S + rs}, r > 1 \tag{4}$$

where *p,q* and S are free parameters of the curve and is the shape factor. A sample of GCS with end points curvature $k0$,= 0, $k1 = 1$ arc length $S = 20$ and shape factor $r = 0.5$ is showed in figure 3. The curvature profile is showed in figure 4.

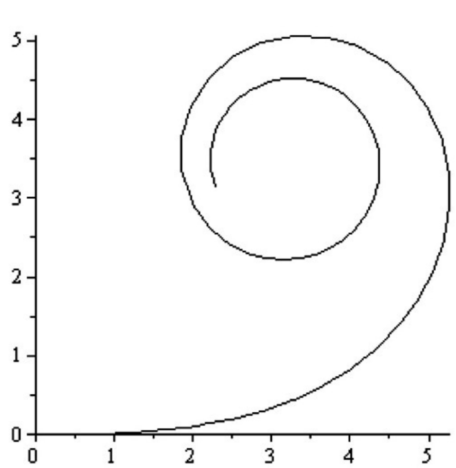


Figure 3. Generalise Cornu spiral GCS

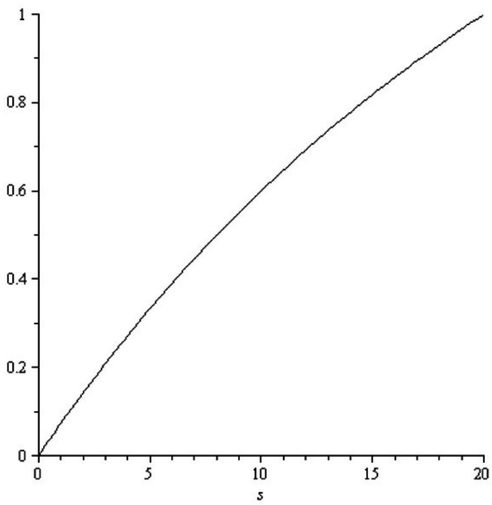


Figure 4. Curvature profile of GCS

When the free parameters in curvature function are set to satisfy specific condition the CGS can be reduced to other forms of curves as below.

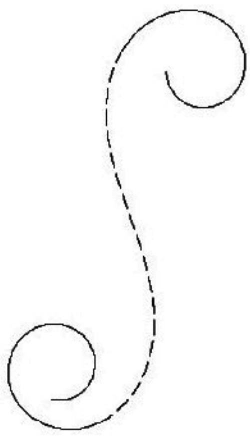
When $q \neq 0$ and $r = 0$ he GCS is reduced to Cornu spiral.
When $q = 0$ and $r \neq 0$ the GCS is reduced to logarithmis spiral.
When $q = 0$ and $r = 0$ the GCS is reduced to circular arc.
When $q = 0$ and $p = 0$ the GCS is reduced to straight line.

APPLICATION OF GCS IN AESTHETIC DESIGN

Since GCS is transcendental function, it has to be approximate by using polynomial in order to apply in the computer aided design (CAD) system. The method of approximating GCS is described in Walton and Meek (2007). In this paper, the segment of GCS is approximated by using piecewise interpolation quintic Hermite curve with second order geometric (G^2). Second order geometric continuity (G^2) refers to continuity of tangent vector and curvature vector. It is an important property in curve design. The application of GCS in aesthetic design is illustrated in vase and glass designs.

Vase

Figure 5a showed the GCS with end point curvatures, $k0 = 1$, $k1 = 1$ arc length $S = 30$ and shape factor r = 0.2. The dash lines represent the segment taken as vase profile. Figure 5b showed the vase profile created using piecewise interpolation of quitic Hermite. The square boxes represent the end points of piecewise Hermite curves. The rendition of vase is showed in figure 6.



5. a

5.b

Figure 5. Vase profile

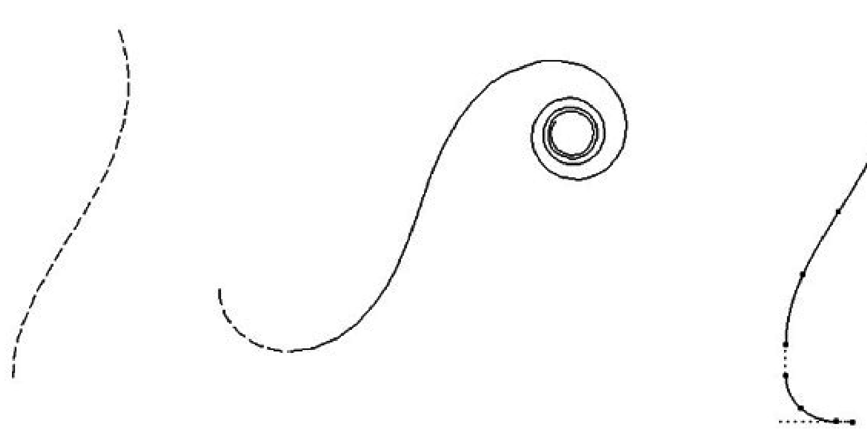
6.a

6.b

Figure 6. Rendition of vase

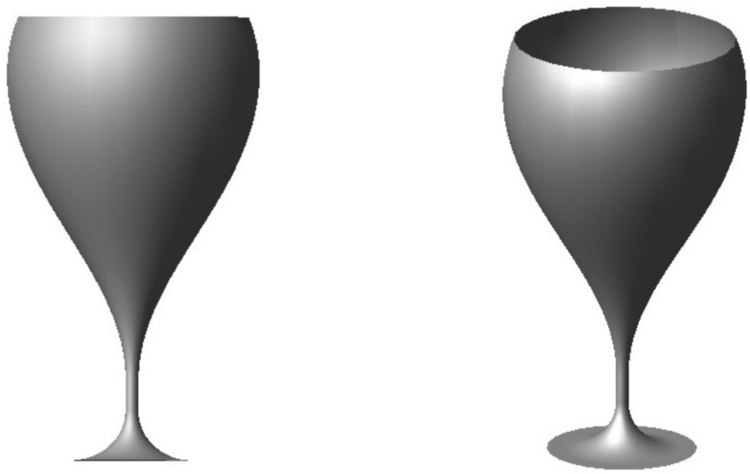
Glass

The curve profile of glass is form by combination of two difference GSC and straight line. Figure 7a showed the GCS with end point curvatures, $k0 = 1$, $k1 = 1$ arc length $S = 3$ and shape factor $r = 0.5$. It is the curve profile of glass body. Figure 7b showed the GCS with end point curvatures $k0 = 3$, $k1 = 8$ arc length $S = 8$ and shape factor $r = 0.2$. The dash lines represent the segment taken as curve profile for glass base. Figure 7c showed the glass profile created by using pricewise interpolation quintic hermite and straight lines. The dotted line represents straight line and the square boxes represent the end points of piecewise Hermite curves. The rendition of glass is showed in figure 8.



7.a 7.b 7.c

Figure 7. Glass profile



8.a 8.b

Figure 8. Rendition of glass.

CONCLUSION

GCS are useful in curve design especially for those aesthetic products which emphasis on fairness of curve. The rational linear curvature properties of GCS guaranty the fairness of the curve. In an addition, the curvature profile can be adjusted through the shape factor in curvature function. The application of GCS is descried with vase and glass design as showed in section 4.

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REFERENCES

Ali, J. M., Tookey, R. M., Ball, J. V., & Ball, A. A. (1999). The generalised Cornu spiral and its application to span generation. *J. Comput. Appl. Math., 102*(1), 37-47.

Burchard *et al*., 1994. H.G. Burchard, J.A. Ayers, W.H. Frey and N.S. Sapidis, Approximation with Aesthetic Constraints. In: N.S. Sapidis, Editor, *Designing Fair Curves and Surfaces*, SIAM, Philadelphia (1994), pp. 3–28.

Farin, G., & Sapidis, N. (1989). Curvature and the fairness of curves and surfaces. *Computer Graphics and Applications, IEEE, 9*(2), 52-57.

Meek, D. S., & Walton, D. J. (1992). Clothoid Spline Transition Spirals. *Mathematics of Computation, 59*(199), 117-133.

McCrae, J., & Singh, K. (2009). Sketching piecewise clothoid curves. *Computers & Graphics, 33*(4), 452-461.

Muratov, S. (2002). The Art of the Violin Design. In S. Muratov (Eds.)

Walton, D. J., & Meek, D. S. (1998). Planar G2 curve design with spiral segments. *Computer-Aided Design, 30*(7), 529-538.

Walton, D. J., & Meek, D. S. (2007). G2 curve design with a pair of Pythagorean Hodograph quintic spiral segments. *Computer Aided Geometric Design, 24*(5), 267-285.